LITERATURE CITED

1. Physics of Thin Films, Vol. 2, Academic Press, New York (1963).
2. G. Parkus, Unsteady Thermal Stresses [in Russian], GIFML, Moscow (1963).
3. A. A. Wells and D. Post, "The dynamic stress distribution surrounding a running crack. A photoelastic analysis," Proc. Soc. Exp. Stress Anal., 16, No. 1 (1958).
4. M. Yoshiki, T. Kanazawa, and H. Itagaki, "An analysis of brittle-fracture propagation," Exp. Mech., 6, 458 (1966).
5. V. M. Finkel', I. A. Kutkin, and V. F. Belorukov, "Branching of cracks in steel," Phys. Met. Metalloved., 15, No. 5 (1963).
6. H. C. van Elst, W. L. Korbee, and C. A. Verbraak, "On the mechanism of propagation of a brittle fracture in steel," Trans. Metallurg. Soc. AIME, 224, 1298 (1962).
7. B. V. Kostrov, Mechanics of the Seismic Center of a Tectonic Earthquake [in Russian], Nauka, Moscow (1975).

INTERACTION BETWEEN CRACKS POSITIONED AT AN ANGLE

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The interaction between cracks in an elastic plane weakened by a system of cracks has, as a rule, been investigated in the case of collinear cracks only. More complex configurations were analyzed in [1, 2], in the first of which four slits placed symmetrically about their common center were studied by using Fourier transforms; in the other, a periodic system of lengthwise–crosswise cracks was studied. In [3] singular integral equations were produced for a system of arbitrarily oriented cracks; numerical results were obtained only for collinear cracks oriented at the same angle to loading direction. In the case of brittle failure the investigation of the interaction between two arbitrarily directed cracks is of interest, this being the subject of the present article.

Let there be two cuts L_1 and L_2 in the xOy plane (Fig. 1) whose parametric equations are (k = 1, 2).

$$L_k: x(t) = a_k t,\ y(t) = b_k t,\ 0 < t_k \leqslant t \leqslant t_{k+2}$$
$$(a_k = \cos\alpha_k,\ b_k = \sin\alpha_k).$$

The boundaries of the cuts are assumed to be stress-free, and at infinity the applied forces are

$$\sigma_x^\infty = \sigma_1, \quad \sigma_y^\infty = \sigma_2, \quad \tau_{xy}^\infty = 0. \tag{1}$$

The following representation [2] of the stress function U is employed:

$$U(x, y) = \frac{1}{2}(\sigma_1 y^2 + \sigma_2 x^2) + \sum_{k=1}^{2} \frac{1}{2\pi} \int_{t_k}^{t_{k+2}} [f_1(t) r_{1k} + f_2(t) r_{2k}] \ln(r_{1k}^2 + r_{2k}^2)\, dt, \tag{2}$$

where

$$r_{1k} = a_k x + b_k y - t;\ r_{2k} = -b_k x + a_k y.$$

The function (2) must satisfy the conditions (1). The conditions on the boundaries of the cracks L_k are

$$(\sigma_y + \sigma_x) + (\sigma_y - \sigma_x)\cos 2\alpha_k - 2\tau_{xy}\sin 2\alpha_k = 0,$$
$$(\sigma_y - \sigma_x)\sin 2\alpha_k + 2\tau_{xy}\cos 2\alpha_k = 0,$$

where

$$\sigma_x = \partial^2 U/\partial y^2;\ \sigma_y = \partial^2 U/\partial x^2;\ \tau_{xy} = -\partial^2 U/\partial x \partial y,$$

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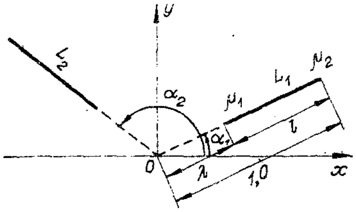

Fig. 1

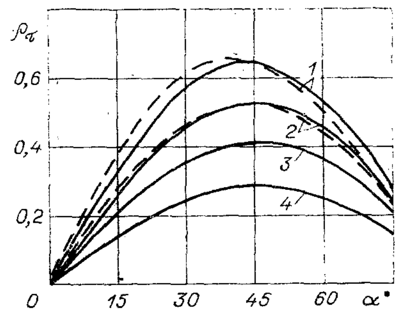

Fig. 2

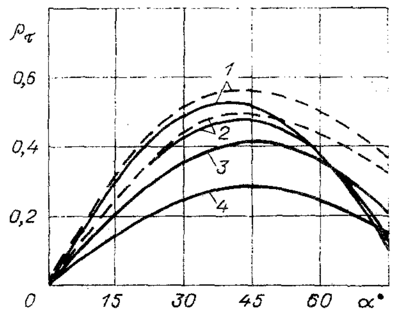

Fig. 3

and hence there follows a system of four singular integral equations for the functions $f_1(t)$ and $f_2(t)$. In the case of symmetrically positioned slits ($\alpha_1 = \alpha$, $\alpha_2 = \pi - \alpha$) which are also of equal length $l = 1 - \lambda$ ($0 < \lambda < 1$), one obtains the equation (k=1, 2).

$$\frac{1}{\pi}\int_{\lambda a}^{a}\left\{\frac{f_k(\xi)}{x-\xi} + f_1(\xi)\,R_{1k}(\xi, x) + f_2(\xi)\,R_{2k}(\xi, x)\right\}d\xi = -p_k, \tag{3}$$

where

$$R_{11} = (cx - \xi)(1 + 2s^2x\xi D)D;$$
$$R_{12} = sx[2\xi(cx - \xi)(x - c\xi)D - 1]D;$$
$$R_{21} = sx[(x - c\xi)^2 - (s\xi)^2]D^2;$$
$$R_{22} = \frac{cx-\xi}{sx}\,R_{21};$$
$$D = (x^2 - 2cx\xi + \xi^2)^{-1};\ c = -\cos 2\alpha,\ s = \sin 2\alpha;$$
$$p_1 = 0.5[(\sigma_1 + \sigma_2) + (\sigma_2 - \sigma_1)\cos 2\alpha];\ p_2 = 0.5(\sigma_1 - \sigma_2)\sin 2\alpha.$$

The transformation

$$\xi = (1/2)[(\xi_2 + \xi_1) + (\xi_2 - \xi_1)v],\quad x = (1/2)[(\xi_2 + \xi_1) + (\xi_2 - \xi_1)u]$$

takes Eqs. (3) to the symmetrical interval (−1, 1). Following [4] one sets

$$f_k(\xi) = g_k(v) = \frac{\sigma g_k^0(v)}{\sqrt{1-v^2}}. \tag{4}$$

The functions $g_k^0(v)$ are found in the form of the interpolatory trigonometric polynomial,

$$g_k^0(v) = \frac{1}{N}\sum_{j=1}^{N}(-1)^{j+1}g_{kj}\sin\vartheta_j\frac{\cos N\vartheta}{\cos\vartheta - \cos\vartheta_j}, \tag{5}$$
$$v = \cos\vartheta,\quad \vartheta_j = \frac{2j-1}{2N}\pi.$$

The problem has thus been reduced to a system of linear algebraic equations for the coefficients g_{kj}. This system must be augmented by a condition that the displacements remain single-valued when travelling around the crack $\int_{\lambda a}^{a} f_k(\xi)\,d\xi = 0$, which by virtue of (5) becomes $\sum_{j=1}^{N} g_{kj} = 0$ (k = 1, 2).

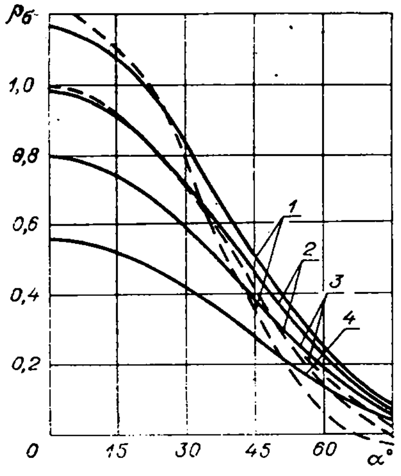

Fig. 4

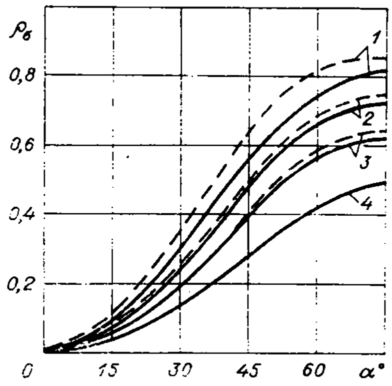

Fig. 5

The stress-intensity coefficients are now determined at both ends of the slits. One has

$$\sigma_x + \sigma_y = 4\operatorname{Re}\Phi(z) = \sigma_1 + \sigma_2 + \sum_{k=1}^{2} \frac{2}{\pi} \int_{t_k}^{t_k+2} [f_1(t) r_{1k} + f_2(t) r_{2k}] \frac{dt}{r_{1k}^2 + r_{2k}^2},$$

and hence one obtains for the Kolosov–Muskhelishvili function the representation

$$\Phi(z) = \frac{\sigma_1 + \sigma_2}{4} + \sum_{k=1}^{2} \frac{1}{2\pi} \int_{t_k}^{t_k+2} \frac{f(t)\, e^{i\alpha_k} dt}{z - e^{i\alpha_k} t}, \quad f = f_1 + if_2. \tag{6}$$

One employs the expansions (4) and (5) and one sets

$$z = x + iy = \frac{1}{2a} e^{i\alpha} [(\xi_2 + \xi_1) + (\xi_2 - \xi_1)\zeta];$$

having evaluated the integral in (6), one obtains

$$\Phi(z) = \frac{1}{N} \sum_{j=1}^{N} (-1)^j (g_{1j} + ig_{2j}) \sin\vartheta_j \frac{\cos(N \arccos\zeta)}{\sqrt{\zeta^2 - 1}\,(\zeta - \cos\vartheta_j)} + R(z), \tag{7}$$

where R(z) is a regular function at the ends.

The extension coefficients in (7) are complex valued, $g_{1j} + ig_{2j}$. Therefore, in the case of the extension of a plane with cracks positioned at an angle one has to take into account the intensity coefficients not only of the normal forces $K_\sigma = \sigma\rho_\sigma$, but also of the tangential forces $K_\tau = \sigma\rho_\tau$. To evaluate these coefficients the limits $2\lim_{M\to M_n} \sqrt{2\pi|MM_n|}\,\Phi(z)$ are evaluated, where at the near ends one has $MM_1 = (\lambda - t)e^{i\alpha}$ $(t < \lambda)$ and at the other ends one has $M_2M = (t-1)e^{i\alpha}$ $(t > 1)$. The expressions for the intensity coefficients at the near ends are

$$\rho_{1\sigma} = \sqrt{\frac{1-\lambda}{2}\pi}\, \frac{1}{N} \sum_{j=1}^{N} (-1)^{j+1} g_{1j} \operatorname{ctg}\frac{\vartheta_j}{2}. \tag{8}$$

The value of $\rho_{1\tau}$ can be obtained from (8) by replacing g_{1j} by g_{2j}. For the far ends in the formula (8) one must replace $\cot(\vartheta_j/2)$ by $-\tan(\vartheta_j\ 2)$, the corresponding expressions now denoted by $\rho_{2\sigma}$, $\rho_{2\tau}$.

In Figs. 2 and 3 the graphs of the intensity coefficients $\rho_{1\tau}$, $\rho_{2\tau}$ are shown in the case of lengthwise extension $(\sigma_2 = 0)$; in Figs. 4 and 5 the graphs of a crosswise extension $\rho_{1\sigma}$, $\rho_{2\sigma}$ $(\sigma_1 = 0)$ are shown as dependent on the slope angle of the slits to the horizontal axis, the value of λ being 0.2, 0.4, 0.6, and 0.8 (curves 1-4, respectively). The solid lines show the intensity coefficients for the far ends and the dashed lines, for the near ends.

One has retained 21 terms in the calculations when solving the algebraic systems for g_{kj}. If one retains 31 terms in the calculations the results remain the same with an error not greater than 10^{-4}.

The intensity coefficients of the tangential forces attain their highest values for the angle between the direction of the cracks and that of the load action equal to 45°. By comparing the values of the intensity coefficients of the tangential stresses with those of normal stresses one observes that for lengthwise extension depending on the values of λ one has the inequality $\rho_\sigma < \rho_\tau$ for angles less than 40–45°. Similar effect is observed for extensions in the crosswise direction though now for angles > 40–45°. For $\alpha = 0$ the computational results agree with the known results [5].

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LITERATURE CITED

1. S. C. Das and I. N. Sneddon, "The stress field near a system of four symmetrically situated line cracks of equal length," in: Mechanics of Continuous Media and Similar Problems of Analysis [in Russian], Nauka, Moscow (1972).
2. L. M. Kurshin and I. D. Suzdal'nitskii, "Stressed state of an elastic plane weakened by infinite sequence of lengthwise–crosswise cracks," Zh. Prikl. Mekh. Tekh. Fiz., No. 5 (1975).
3. A. P. Dashitsin and M. P. Savruk, "Integral equations of the planar problem in the theory of cracks," Prikl. Mat. Mekh., 38, No. 4 (1974).
4. A. I. Kalandiya, Mathematical Methods of Two-Dimensional Elasticity [in Russian], Nauka, Moscow (1973).
5. V. V. Panasyuk, The Equilibrium Limit of Brittle Bodies with Cracks [in Russian], Naukova Dumka, Kiev (1968).

DETERMINATION OF THE RATE OF EVOLUTION OF ELASTIC ENERGY FOR A Γ-SHAPED CRACK BY THE METHOD OF MEASUREMENT OF THE PLIABILITY

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At the present time, the values of the stress-intensity coefficients and the rate of evolution of elastic energy are well known for various configurations of a body with rectilinear cracks [1]. For a nonrectilinear crack, the only known problem is that of an arc-shaped crack in an infinite sheet with homogeneous elongation in an arbitrary direction [2].

To clarify the character of the propagation of cracks in laminar materials, it is important to know the rate of evolution of elastic energy for Γ-shaped cracks, where, after the breakdown of an element of the matrix, peeling-off starts in the composite material. The theoretical solution of such a problem is rather complex and, up to the present time, has not been carried through. In the present article, this problem is solved experimentally by the method of measurement of the pliability.

A method for determining the rate of evolution of elastic energy from the change in the pliability with an increase in the length of the crack was proposed a long while ago [3]; however, we know of no work where the method has been implemented in practice. This is obviously connected with the necessity of making extremely exact measurements, which is difficult to do practically. In the present work, the method of measuring the frequency of the intrinsic vibrations was used, which makes it possible to measure the pliability of a sample with an accuracy up to 0.05%.

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